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Failure-based sealing reliability analysis considering dynamic interval and hybrid uncertainties

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Highlights

- The degradation with a small sample is modelled using a dynamic interval process.
- The reliability model with hybrid uncertainty is proposed.
- The case study of sealing reliability prediction and evaluation is presented.

Abstract

In the reliability analysis of a sealing structure, radial clearance of the contact surface is usually regarded as a failure criterion, and the sample size is usually quite small, which brings great challenges to uncertainty quantification. Therefore, this paper proposes a reliability analysis method based on the leakage mechanism of the sealing. With the application of dynamic interval, the proposed method can be used to deal with problem of degradation in small sample to evaluate reliability. Moreover, the dynamic reliability with the mixture of the probabilistic and non-probabilistic variables can be obtained using the proposed method. An illustrative numerical case study of a spool valve is conducted in order to validate the proposed method and the implemented reliability sensitivity analysis. The proposed method is of great help in evaluating and predicting reliability with small degradation sample and hybrid uncertainties.

Keywords

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reliability analysis, dynamic interval, hybrid uncertainty, physics of failure, sealing.

1. Introduction

The problem of machines and devices assessment is considered as one of the most important and relevant reliability analysis issues [12, 13, 27, 29]. It is directly related to many aspects of technical systems exploitation, including efficiency and sustainability dimensions [16, 18, 19, 22]. The exploitation assessment is also a key component of an operational decision-making process as a result of the established maintenance policy [28, 37]. One of the most important features underlying the construction of the exploitation assessment models is reliability. In recent years, more and more engineers and statisticians have acquired and processed degradation data through the measurement of performance parameters of several products in order to predict their reliability. Mathematical and other solutions built on the basis of the reliability methodology are still up-to-date [22, 41], especially when it is necessary to take into account a particular degree of uncertainty [22, 38, 46].

The key element to increase the reliability and performance of mechanical devices is the structural reliability analysis. The contemporary complexity of machines and devices still makes it a big challenge for both scientists and practitioners. Because of a very large number of calculations required for assessing small failure probabilities, this

is a labor-intensive and time-consuming process [30]. Design and operational parameters of mechanical elements of devices due to the effects of environmental changes are often uncertain. According to [10], different typologies of uncertainty and analysis, methods for reliability can be divided into two main categories: time-variant and time invariant methods. Therefore, the analysis of machines and devices reliability is focused on the identification and evaluation of various types of uncertainty, their effects and the assessment of the probability of a component failure [38].

These research challenges appear when spool valves are taken into consideration as an example of a mechanical element. Spool valve is a basic part in a hydraulic system, where reliability has a significant influence on the entire system [35]. The reciprocating sliding operations result in the inevitable wearing of a spool and sleeve, which leads to leakage in the sealing and eventually causes the failure of the sealing [25]. The wear degradation was investigated by Liu et al [26] and Yang et al [43]. In these studies, the failure is caused by the wear volume exceeding the threshold. Various degradation models have attracted the attention of researchers all over the world [33]. Gorjian et al [15] and Shahraki et al [32] reviewed various degradation models in a reliability analysis. Moreover, in the work [8], a probabilistic method based on a stochastic differential calculation for the reliability

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assessment of structural components is defined. Andrieu-Renaud et al [1] developed a method known as PHI2, based on a cross approach that solves reliability problems using classic time-invariant reliability tools. In the aspect of a modern performance-based design, Au and Beck [2] implemented Subset Simulation (SS) to evaluate the performance of structures. The SS method, for the assessment of small failure probabilities, was also used by Bourinet et al [5] in the approach referred as 2SMART.

As a matter of fact, the failure of the sealing of a spool valve is caused by internal leakage [31], whose mechanism shall be taken into account in order to conduct the reliability analysis [11]. The leakage between contacting surfaces in valves is likely to be influenced by the direction of the surface anisotropy or lay [3, 23]. A quantitative multi-scale analysis of the surface morphology or curvature are considered as valuable tools for elucidating changes in the anisotropy caused by processing, and for the performance indication including sealing, lubrication, and friction [4].

The existence of degradation brings the issue of time-variant reliability [25], which indicates that the reliability of the seal varies with a task. In practice, the wear data is usually hard to measure [1] and the observed data is often insufficient to quantify the uncertainty using a probabilistic approach [42]. Therefore, the interval method is applied to deal with the problem of a small sample [20]. In order to solve this problem, Liao et al [24] built a reliability model of aviation seal with an interval method. The interval method is able to quantify the uncertainty with limited data, and has been eagerly widely applied in the field of epistemic uncertainty and non-probabilistic reliability. Kang et al [21] proposed belief reliability as a reliability metric under the epistemic uncertainty. You et al [45] presented a novel structural reliability analysis method with fuzzy random variables. Besides, more than one type of uncertainties exist due to the interval and random variables [36]. Hence, the reliability modelling with the hybrid uncertainty has become another research focus in recent years [6, 17]. Moreover, Wang [39], according to [40, 47], defined a hybrid reliability analysis (HRA) as a task that quantifies two types of uncertainties, and as a core one in the structural reliability research. Chakraborty et al [7] analyzed structural probabilistic safety under the hybrid uncertainty. Sun et al [34] built the time-variant reliability model using the hybrid non-probability method. Jiang et al [20] reviewed several main research directions in the probability-interval hybrid uncertainty analysis, and provided an outlook for the potential research aspects. A Bayesian approach for a sealing failure analysis was presented in [44], where the radial clearance height was regarded as a failure criterion, and where the observed data was assumed to follow the gamma distribution.

However, in the reliability analysis of the sealing, a performance function shall be established based on a leakage mechanism. The sample of degradation data is usually very small, and the existing research cannot solve the problem of a small sample with degradation. Therefore, this paper proposed a physics of a failure-based reliability analysis model, where the dynamic interval is applied to deal with the issue of degradation in a small sample. The proposed method takes a failure mechanism into consideration to build a dynamic reliability model, and the reliability is resolved with the hybrid uncertainty method. Moreover, an illustrative case study of the sealing in a spool valve is conducted to validate the proposed method and to analyse the dynamic reliability of the sealing.

2. Dynamic reliability model considering degradation and hybrid uncertainty

For the need of the dynamic reliability model design, it's necessary to introduce an uncertainty process and, next, a dynamic interval process. The uncertainty process can be expressed as:

$$\{X(t_i) \in X^I(t), t \in T\} \quad (1)$$

where: $X(t_i)$ denotes an interval value at a given time t_i . For the time t_1, t_2, \dots, t_n , the joint distribution region composed of interval variables is a hypercube domain. Therefore, the uncertainty process is defined as a dynamic interval process [38].

The dynamic interval can be described as a time-variant interval process where an interval changes with time. For the given interval process $X(t)$, $\overline{X(t)}$ and $\underline{X(t)}$ denote the upper and lower limits, respectively, and the mean function of the interval process is expressed as:

$$X^c = \frac{\overline{X(t)} + X(t)}{2} \quad (2)$$

and the radius function is expressed as:

$$X^r = \frac{\overline{X(t)} - X(t)}{2} \quad (3)$$

Once X^r and X^c are obtained, the uncertainty characteristics of each specific moment can be determined. The mean and radius functions can be obtained using the fitting methods, such as a linear model, an exponential model or a stochastic process model. The dynamic interval process requires the interval information at each observation point, which enables the proposed method to appraise the dynamic reliability with very limited data. Furthermore, the fitted curve with the observation can be used to predict the reliability in a longer period.

When both the random uncertainty and interval uncertainty exist at the same time in a structure, the performance function can be expressed as:

$$Z = g(X, Y) \quad (4)$$

where: $g(\cdot)$ denotes the function of X and Y ,

$X = \{X_1, X_2, \dots, X_n\}$ denotes independent n -dimensional interval vector,

$X_i \in X_i^I = [X_i^L, X_i^U], i = 1, 2, \dots, n$,

$Y = \{Y_1, Y_2, \dots, Y_m\}$ denotes independent m -dimensional random vector.

The structure is considered as reliable with $Z > 0$, and unreliable with $Z < 0$. In practice, Z is the difference between the performance threshold and performance parameters, which can be expressed as:

$$Z = P_{th} - P(x_1, x_2, \dots, x_n) \quad (5)$$

where: $P(x_1, x_2, \dots, x_n)$ is the function of critical performance parameters modelled with a failure mechanism of a product, P_{th} denotes the performance threshold.

The reliability is expressed as:

$$R = P\{g(X, Y) > 0\} \quad (6)$$

The reliability result becomes an interval instead of a probability value due to the existence of interval vectors, of which the lower limit R_L and upper limit R_U are given by the following equations:

$$R_L = P\{\min g(X, Y) > 0\} \quad (7)$$

$$R_U = P\{\max g(X, Y) > 0\}$$

When interval vectors or random vectors are time-dependent, the reliability given in (7) becomes a time-variant interval with $R_L(t)$ and $R_U(t)$. In the proposed model, the leakage mechanism of the sealing in a spool valve with necessary features is presented to determine the reliability.

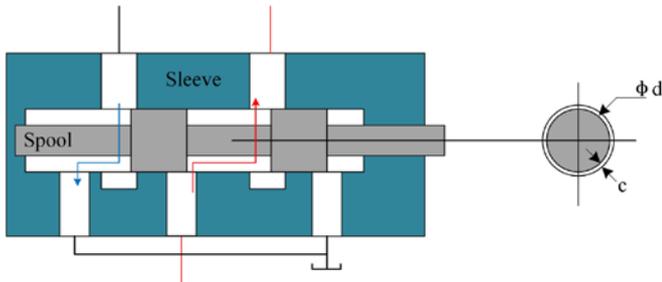


Fig. 1. Schema of a spool valve and clearance [44]

A typical directional valve is shown in Fig. 1. As discussed in [44], an internal leakage is given by the following equation:

$$Q = \Delta P \frac{\pi d c^3}{12 \mu L} \quad (8)$$

where: Q - internal leakage of a spool valve,

ΔP - pressure difference,

d - diameter of a spool valve,

c - radial clearance height,

μ - dynamic viscosity of hydraulic oil,

L - clearance length.

The high frequency of the back and forth sliding movements cause the wear of spools and sleeves [14]. The wear will finally cause the increase of the clearance and internal leakage exceeding its allowable threshold. Thus, the sealing of the spool valve is regarded as a failure. Therefore, according to formula (5), the performance function of the sealing is given by:

$$Z = Q_{th} - \Delta P \frac{\pi d c^3}{12 \mu L} \quad (9)$$

where: Q_{th} - the threshold of leakage.

For each observation, the maximum and minimum are regarded as interval limits:

$$c_{n_i}^I = [c^L, c^U] = [\min(S(i)), \max(S(i))] \quad (10)$$

where: $S(i)$ - the samples from the i_{th} observation,

c^L, c^U - interval limits.

Based on the performance function in (9), the reliability can be calculated using the first order second moment (FOSM) method. In this circumstance, Z is the function of $Q_{th}, \Delta P, d, \mu, L, c$. Z varies with the stroke, which is expressed as:

$$Z(n) = g(Q_{th}, \Delta P, d, \mu, L, c(n)) \quad (11)$$

In formula (11), $c(n)$ varies with the stroke and is modelled with a dynamic interval process. The reliability decreases monotonically with the increase of c . Therefore, the dynamic reliability interval of the sealing can be obtained using the following formulas:

$$R^L(n) = P\{g(Q_{th}, \Delta P, d, \mu, L, c) > 0 | c = c^U(n)\} \quad (12)$$

$$R^U(n) = P\{g(Q_{th}, \Delta P, d, \mu, L, c) > 0 | c = c^L(n)\}$$

3. Numerical example

In the case study, the variables $Q_{th}, \Delta P, d, \mu, L$ (9) are regarded as random variables, and the distributions of the random variables are assumed to be normal, as shown in Table 1. The clearance of the spool and sleeve c is usually measurable with a small sample which is regarded as an interval variable and described with a dynamic interval process.

For the purposes of determining the dynamic interval of clearance, the wear volumes are observed per 50,000 strokes. The clearance of a

Table 1. Random variables in sealing

Parameter	Type of distribution	Mean	Coefficient of variation
Q_{th}	Normal distribution	14.54 ml/min	0.03
ΔP	Normal distribution	27.50 MPa	0.03
d	Normal distribution	7.60 mm	0.03
μ	Normal distribution	0.013 kg/(m·s)	0.03
L	Normal distribution	0.50 mm	0.03

Table 2. Wear volumes and interval of clearance[44]

Unit	Number of Strokes (10 thousands)																			
	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
Sample1	9	13	25	28	31	35	41	45	49	56	61	63	69	76	81	85	92	96	104	113
Sample2	5	10	17	24	30	39	44	51	58	63	68	76	80	85	93	99	104	108	113	119
Sample3	6	12	18	23	26	30	38	43	49	57	63	70	74	77	84	92	99	101	107	113
Sample4	3	6	17	20	25	33	41	46	54	60	64	69	72	83	89	93	96	100	104	111
Sample5	10	15	19	24	34	43	49	54	61	65	70	74	77	82	84	91	99	103	108	115
c^U	10	15	25	28	34	43	49	54	61	65	70	76	80	85	93	99	104	108	113	119
c^L	3	6	17	20	25	30	38	43	49	56	61	63	69	76	81	85	92	96	104	111

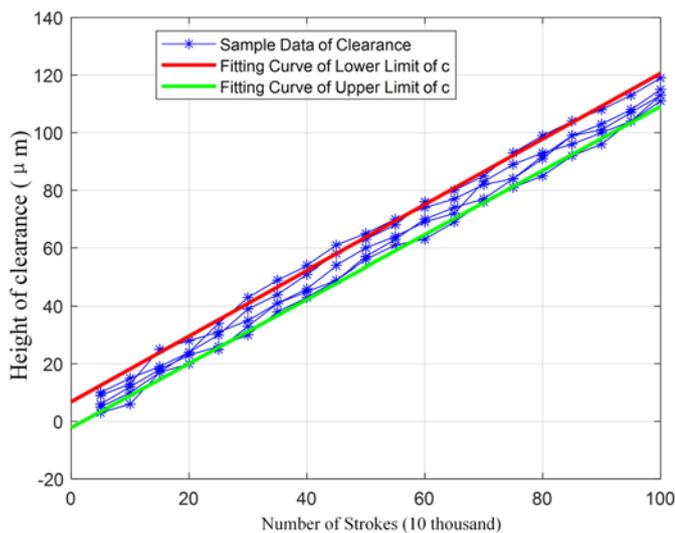


Fig. 2. Sample data of clearance and fitting limits

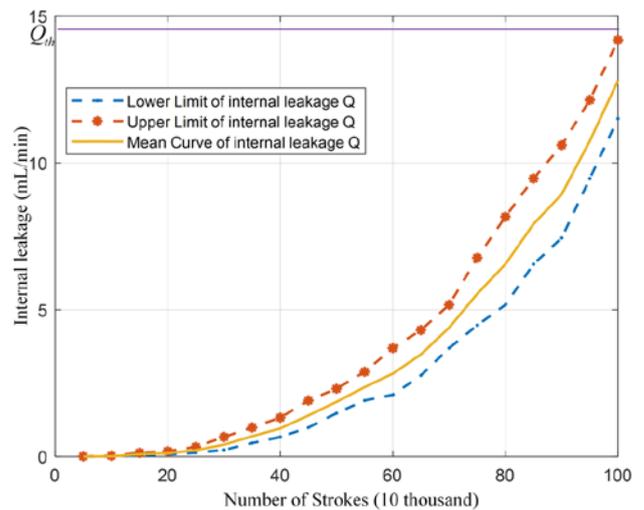


Fig. 3. The trend of internal leakage

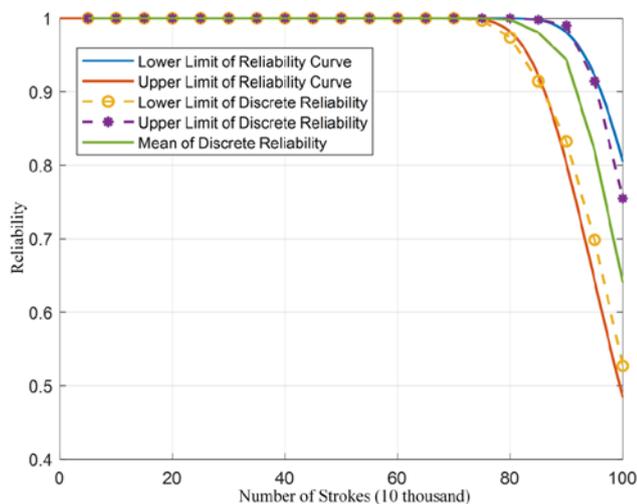


Fig. 4. Dynamic reliability of the sealing varies with the stroke

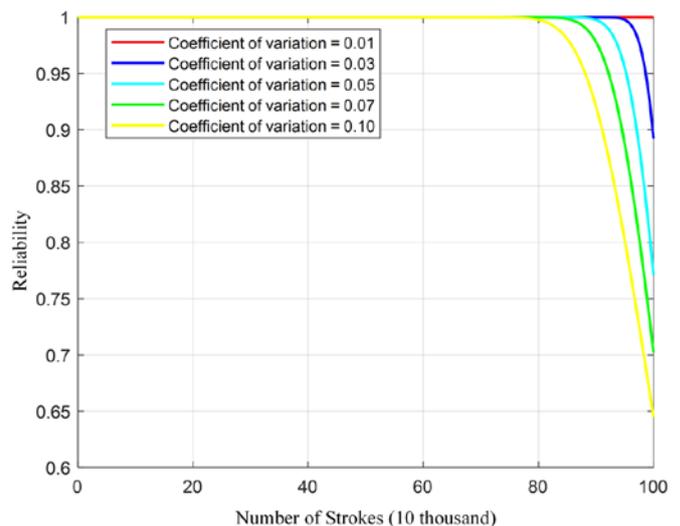


Fig. 5. Dynamic reliability with various coefficients of the variation

spool valve of 5 samples is listed in Table 2, followed by the interval limits of c^L and c^U (10).

With the interval data of clearance, the trend of the upper and lower limits can be obtained using the curve fitting method, such as the least square method. The functions of c^L and c^U vary with the strokes n and are fitting curves as:

$$\begin{aligned} c^L(n) &= 1.113n - 2.179 \\ c^U(n) &= 1.138n + 6.811 \end{aligned} \quad (12)$$

where n denotes the number of strokes with a unit of 10 thousand. The initial sample data and the fitting curve of limits are depicted in Fig. 2.

With the increase of clearance, the internal leakage also ascends with the strokes. The leakage also becomes an interval variable due to the interval uncertainty of clearance. The limits and mean curves are depicted in Fig. 3. With the stroke of the spool valve increase, the internal leakage gets closer to a failure criterion, which will cause the descending of the sealing reliability.

The sealing reliability from the discrete interval and fitting dynamic interval of c are depicted according to (12) in Fig. 4.

It can be concluded from the performance function that $\Delta P, d$ and c have a negative effect on the reliability of the sealing, while Q_{th}, μ and L have a positive effect. In addition, the variation of all the parameters in the spool valve will also influence the reliability trend of the sealing.

In order to compare the differences caused by the variation of the parameters, dynamic curves with various coefficients of the variation are depicted in Fig. 5. It can be seen from the curves in Fig. 5 that with the increase of the variation, the reliability of the sealing will decrease earlier with a lower terminal point. Therefore, it is suggested that the inconsistency and uncertainty shall be reduced to obtain a longer lifetime with higher reliability. It can be seen from the curves in Fig. 5 that with the increase of the variation, the reliability of the sealing will decrease earlier with a lower terminal point. Therefore, it is suggested that the inconsistency and uncertainty shall be reduced to obtain a longer lifetime with higher reliability.

As it was mentioned above, the proposed method enables the reliability prediction with less data. Therefore, the first 15 observed data in Table 2 was used to build a dynamic reliability model with the proposed method. Additionally, the reliability curves with less observations and full observations are depicted in Fig. 6.

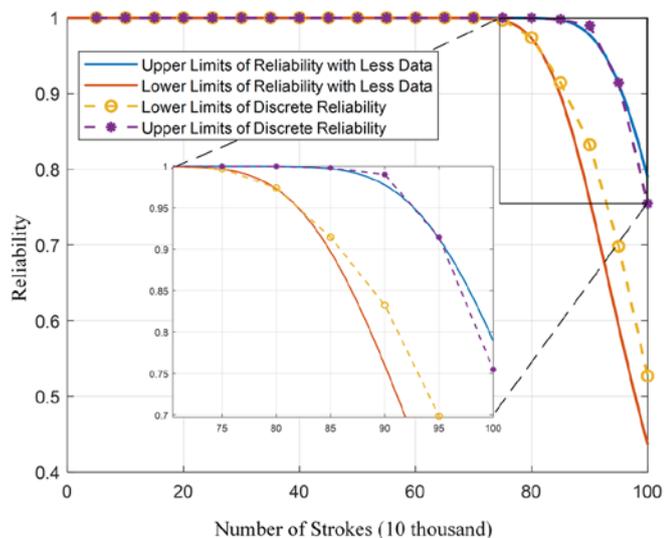


Fig. 6. Comparison between the predicted reliability curves and the evaluated reliability value

Table 3. The predicted and evaluated reliability

Items	Number of Strokes (10 thousand)				
	80	85	90	95	100
Predicted upper limits	0.9998	0.9970	0.9774	0.9125	0.7894
Evaluated upper limits	0.9999	0.9980	0.9899	0.9143	0.7548
Relative Error	0.01%	0.10%	1.2%	0.19%	4.5%
Predicted lower limits	0.9726	0.8968	0.7598	0.5931	0.4366
Evaluated lower limits	0.9739	0.9143	0.8323	0.6984	0.5270
Relative Error	0.13%	1.9%	8.7%	15%	17%

As shown in Fig. 6, the predicted reliability is basically consistent with the evaluated reliability. The numerical comparison results are listed in Table 3.

The last 5 groups of the data (observed at 80 to 100) in Table 2 are used to verify the proposed method. The relative error at each observation point is very small. It can be seen from the comparison in Table 3 that the predicted reliability limits with partial data are quite close to the evaluated limits with full data, revealing that the proposed method is very efficient in predicting the reliability limits with the insufficient observed data.

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4. Conclusions

This paper proposed a failure-based reliability analysis method for sealing. In the reliability analysis of sealing, the allowable leakage is regarded as a failure criterion. The proposed method takes a failure mechanism of sealing into consideration and establishes a performance function of sealing with an explicit expression with which the sensitivity of different variables can be easily obtained. Besides, a dynamic interval is adapted to deal with the issues of a small sample degradation. The reliability can be evaluated with the hybrid uncertainty method. The obtained reliability result becomes two boundary curves instead of one reliability curve due to the existence of both interval variables and random variables.

The proposed method can be used to predict the wear interval in sealing with small sample and hybrid uncertainties, and the reliability can be evaluated and predicted with limited observation data. The operators can make dependable maintenance decision with the reliability trend curve, thus replacing the sealing when reliability drops to a certain level.

Moreover, an illustrative case study is conducted to verify the proposed method, where the data in [44] is applied to build a dynamic reliability model. The method proposed in this paper is more concise and confident under the circumstance of only a small amount

of data than that with the gamma process and Bayesian estimation. Furthermore, it is verified if the proposed method can be used to predict the reliability with high precision in case of partial data, as well as to evaluate the reliability with a reasonable interval result in case of full data. The following can be concluded from the previous discussions:

- The proposed method can deal with the problem of degradation with a small sample, which is common in many engineering practices. The dynamic interval process can be applied to quantify the dynamic uncertainty with insufficient data.
- A failure mechanism shall be taken into account when establishing the performance function. The influences and sensitivity of different parameters can be obtained easily with an explicit expression.
- The hybrid uncertainty problem is transformed into the probabilistic reliability in the proposed method, with which the reliability boundary curves can be obtained.
- Compared with the approach of the reliability analysis in [44], the proposed method has the advantage of simplicity and credibility without any subjective hypothesis for the observed data.

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